Chapter 6

Time Value of Money

Concepts

Time Value of Money

Interest is the rent paid for the use of money over time.

That's right! A dollar today is more valuable than a dollar to be received in one year.

Simple Interest

Interest amount = \( P \times i \times n \)

Assume you invest $1,000 at 6% simple interest for 3 years.

You would earn $180 interest.

\( ($1,000 \times .06 \times 3 = $180) \)

(or $60 each year for 3 years)
When we compound interest, we assume you earn interest on both principal and interest.

<table>
<thead>
<tr>
<th>Principal</th>
<th>Interest</th>
</tr>
</thead>
</table>

Assume we will save $1,000 for three years and earn 6% interest compounded annually.

What is the balance in our account at the end of three years?

<table>
<thead>
<tr>
<th>Original balance</th>
<th>$1,000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>First year interest</td>
<td>$60.00</td>
</tr>
<tr>
<td>Balance, end of year 1</td>
<td>$1,060.00</td>
</tr>
<tr>
<td>Balance, beginning of year 2</td>
<td>$1,060.00</td>
</tr>
<tr>
<td>Second year interest</td>
<td>$63.60</td>
</tr>
<tr>
<td>Balance, end of year 2</td>
<td>$1,123.60</td>
</tr>
<tr>
<td>Balance, beginning of year 3</td>
<td>$1,123.60</td>
</tr>
<tr>
<td>Third year interest</td>
<td>$67.42</td>
</tr>
<tr>
<td>Balance, end of year 3</td>
<td>$1,191.02</td>
</tr>
</tbody>
</table>
Future Value of a Single Amount

Multiply a year’s beginning principal by the interest rate and add that year’s interest to the account balance.

\[
\begin{align*}
$1,000.00 \times 1.06 & = $1,060.00 \\
\text{and} \\
$1,060.00 \times 1.06 & = $1,123.60 \\
\text{and} \\
$1,123.60 \times 1.06 & = $1,191.02
\end{align*}
\]

Writing in a more efficient way, we can say . . .

\[
$1,000 \times 1.06 \times 1.06 \times 1.06 = $1,191.02
\]

or

\[
$1,000 \times (1.06)^3 = $1,191.02
\]

We can generalize this as . . .

\[
FV = PV \ (1 + i)^n
\]
Find the **Future Value** of $1 table in your textbook.

Find the factor for 6% and 3 periods.

Solve our problem like this...

\[
FV = 1,000.00 \times 1.19102 \\
FV = 1,191.02
\]

You invest $10,000 today and earn 8% interest for 8 years. What will the balance in your account be at the end of 8 years if...

A. Interest is simple. 
B. Interest is compounded annually.
A - Simple Interest

$10,000 \times 0.08 \times 8 = $6,400

$10,000 + $6,400 = $16,400

B - Compound Annually

$10,000 \times 1.85093 = $18,509.30

Future Value of a Single Amount

Present Value of a Single Amount

Instead of asking what is the future value of a current amount, we might want to know what amount we must invest today to accumulate a known future amount.

This is a present value question.

Remember our equation?

\[ FV = PV \times \left(1 + i\right)^n \]

We can solve for PV and get . . . .

\[ PV = \frac{FV}{\left(1 + i\right)^n} \]
We can rearrange the equation . . .

\[ PV = FV \frac{1}{(1 + i)^n} \]

or

\[ PV = FV (1 + i)^n \]

Find the Present Value of $1 table in your textbook.

Assume you plan to buy a new car in 5 years and you think it will cost $20,000 at that time.

What amount must you invest today in order to accumulate $20,000 in 5 years, if you can earn 8% interest compounded annually?
Present Value of a Single Amount

\[ i = .08, \quad n = 5 \]

Present Value Factor = .68058

\[ \$20,000 \times .68058 = \$13,611.60 \]

If you deposit $13,611.60 now, at 8% annual interest, you will have $20,000 at the end of 5 years.

Time Value of Money Example

If you deposit $5,000 in a bank at 8% interest compounded annually, how much will you have in 5 years? . . . in 10 years?

<table>
<thead>
<tr>
<th></th>
<th>5 Years</th>
<th>10 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>$7,387</td>
<td>$8,144</td>
</tr>
<tr>
<td>b.</td>
<td>$7,347</td>
<td>$10,795</td>
</tr>
<tr>
<td>c.</td>
<td>$7,347</td>
<td>$9,471</td>
</tr>
<tr>
<td>d.</td>
<td>$6,984</td>
<td>$9,186</td>
</tr>
</tbody>
</table>

Future Value of $1 Table

\[ \$5,000 \times 1.46933 = \$7,346.65 \]
\[ \$5,000 \times 2.15892 = \$10,794.60 \]
What amount must you deposit today at 6% interest compounded annually, to have $10,000 for your first year of college 5 years from now?

a. $7,462  
b. $7,921  
c. $7,473  
d. $7,581

Present Value of $1 Table

$10,000 \times 0.74726 = 7,472.60$

On June 1, your company purchases equipment by paying $5,000 down and issuing a $27,000 noninterest-bearing note payable that is due in three years. Similar transactions carry a stated interest rate of 6%.

What is the purchase price of the equipment?
Present Value

Face of note $27,000
Present value of $1 (i = 6%, n = 3) × 0.83962 $22,670
Present value of note $22,670
Cash paid 5,000
Cost of equipment $27,670

Journal entry to record the note and equipment

GENERAL JOURNAL

<table>
<thead>
<tr>
<th>Date</th>
<th>Description</th>
<th>Freq. Ref.</th>
<th>Debit</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jun. 1</td>
<td>Equipment</td>
<td></td>
<td>27,670</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Discount on Notes Payable</td>
<td></td>
<td>4,330</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Notes Payable</td>
<td></td>
<td>27,000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cash</td>
<td></td>
<td>5,000</td>
<td></td>
</tr>
</tbody>
</table>

Solving for Other Values

\[ FV = PV \times (1 + i)^n \]

There are four variables needed when determining the time value of money.
If you know any three of these, the fourth can be determined.
Solving for Other Variables
Example

Suppose a friend wants to borrow $1,000 today and promises to repay you $1,092 two years from now. What is the annual interest rate you would be agreeing to?

a. 3.5%
b. 4.0%
c. 4.5%
d. 5.0%

No Explicit Interest

Some notes do not include a stated interest rate. We call these notes noninterest-bearing notes.

Even though the agreement states it is a noninterest-bearing note, the note does, in fact, include interest.

We impute an appropriate interest rate for a loan of this type to use as the interest rate.
The objective of valuing an asset or liability using present value is to approximate the fair value of that asset or liability.

Expected Cash Flow Approach

Expected Cash Flow
× Risk-Free Rate of Interest
= Present Value

Basic Annuities

An annuity is a series of equal periodic payments.

Ordinary Annuity

An annuity with payments at the end of the period is known as an ordinary annuity.
An annuity with payments at the beginning of the period is known as an annuity due.

Future Value of an Ordinary Annuity

The equation to find the future value of an annuity is...

\[ FVA = \frac{(1 + i)^n}{i} - 1 \]

Because this is the equation for the FV of $1, the equation for FVA should be easy to remember.

Future Value of an Ordinary Annuity

To find the future value of an ordinary annuity, multiply the amount of a single payment or receipt by the future value factor.
We plan to invest $2,500 at the end of each of the next 10 years. We can earn 8%, compounded annually, on all invested funds.

What will be the fund balance at the end of 10 years?

Future Value of an Ordinary Annuity

Amount of annuity $2,500.00

Future value of ordinary annuity of $1

\( (i = 8\%, n = 10) \times 14.4866 \)

Balance $36,216.50

Find the Future Value of Ordinary Annuity of $1 factor in your textbook.

Future Value of an Annuity Due

Compute the future value of $10,000 received at the beginning of each of the next four years with interest at 6% compounded annually.
Future Value of an Annuity Due

Compute the future value of $10,000 received at the beginning of each of the next four years with interest at 6% compounded annually.

<table>
<thead>
<tr>
<th>Amount of annuity</th>
<th>$10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>FV of annuity due of $1</td>
<td>(i=6%, \ n=4) \times 4.63710</td>
</tr>
<tr>
<td>Present value of annuity</td>
<td>$46,371</td>
</tr>
</tbody>
</table>

Present Value of an Ordinary Annuity

The equation to find the present value of a series of $1 payments is . . . .

\[
PV = \frac{1}{i} \left( \frac{1}{(1 + i)^n} \right)
\]

This is the equation for the PV of $1
You wish to withdraw $10,000 at the end of each of the next 4 years from a bank account that pays 10% interest compounded annually.

How much do you need to invest today to meet this goal?

Present Value of an Ordinary Annuity

<table>
<thead>
<tr>
<th>PV</th>
<th>Annuity</th>
<th>Factor</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV1</td>
<td>$10,000</td>
<td>0.90909</td>
<td>$9,090.90</td>
</tr>
<tr>
<td>PV2</td>
<td>10,000</td>
<td>0.82645</td>
<td>8,264.50</td>
</tr>
<tr>
<td>PV3</td>
<td>10,000</td>
<td>0.75131</td>
<td>7,513.10</td>
</tr>
<tr>
<td>PV4</td>
<td>10,000</td>
<td>0.68301</td>
<td>6,830.10</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>$31,698.60</td>
</tr>
</tbody>
</table>

If you invest $31,698.60 today you will be able to withdraw $10,000 at the end of each of the next four years.
Present Value of an Ordinary Annuity

<table>
<thead>
<tr>
<th>Annuity</th>
<th>PV Factor</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV1 $10,000</td>
<td>0.909099</td>
<td>$9,090.90</td>
</tr>
<tr>
<td>PV2 10,000</td>
<td>0.826458</td>
<td>8,264.50</td>
</tr>
<tr>
<td>PV3 10,000</td>
<td>0.751311</td>
<td>7,513.10</td>
</tr>
<tr>
<td>PV4 10,000</td>
<td>0.683016</td>
<td>6,830.10</td>
</tr>
<tr>
<td>Total</td>
<td>3.16986</td>
<td>$31,698.60</td>
</tr>
</tbody>
</table>

Can you find this value in the Present Value of Ordinary Annuity of $1 table?

---

How much must a person 65 years old invest today at 8% interest compounded annually to provide for an annuity of $20,000 at the end of each of the next 15 years?

- a. $153,981
- b. $171,190
- c. $167,324
- d. $174,680

**PV of Ordinary Annuity $1**

- Payment: $20,000.00
- PV Factor: 8.55948
- Amount: $171,189.60
Present Value of an Ordinary Annuity

Assume the person only has $140,000. What annuity will this amount provide at the end of each of the next 15 years if it is invested today at 8% interest compounded annually?

a. $15,891
b. $16,356
c. $17,742
d. $18,123

Present Value of an Ordinary Annuity

Assume the person only has $140,000. What annuity will this amount provide at the end of each of the next 15 years if it is invested today at 8% interest compounded annually?

a. $15,891
b. $16,356
c. $17,742
d. $18,123

Present Value of an Ordinary Annuity

Assume the person only has $140,000. What annuity will this amount provide at the end of each of the next 15 years if it is invested today at 8% interest compounded annually?

a. $15,891
b. $16,356
c. $17,742
d. $18,123

Present Value of an Annuity Due

Compute the present value of $10,000 received at the beginning of each of the next four years with interest at 6% compounded annually.
Compute the present value of $10,000 received at the beginning of each of the next four years with interest at 6% compounded annually.

\[
\begin{align*}
\text{Amount of annuity} & \quad 10,000 \\
\text{PV of annuity due of $1} & \quad (i=6\%, \, n=4) \\
& \quad \times \, 3.67301 \\
\text{Present value of annuity} & \quad 36,730 \\
\end{align*}
\]

Present Value of an Annuity Due

Western Gas, Inc. lost a lawsuit requiring the company to pay $2,250,000 immediately or $260,000 ($3,900,000 total) at the end of each of the next 15 years. Assume Western Gas can earn 9% on all funds available.

Which settlement option would you recommend?

Present Value of Annuities

\[
\begin{align*}
\text{Annual payment} & \quad 260,000 \\
\text{PV factor for ordinary annuity, } n = 15, \, i = 9\% & \quad \times \, 8.06069 \\
\text{PV of annuity payments} & \quad 2,095,779 \\
\end{align*}
\]

Because the present value of the payments is less than the lump sum payment, you would recommend that Western Gas make the annual payments of $260,000.
In a deferred annuity, the first cash flow is expected to occur more than one period after the date of the agreement.

Present Value of a Deferred Annuity

On January 1, 2003, you are considering an investment that will pay $12,500 a year for 2 years beginning on December 31, 2005. If you require a 12% return on your investments, how much are you willing to pay for this investment?

<table>
<thead>
<tr>
<th>Cost Today?</th>
<th>1/1/03</th>
<th>12/31/03</th>
<th>12/31/04</th>
<th>12/31/05</th>
<th>12/31/06</th>
<th>12/31/07</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12,500</td>
<td>$12,500</td>
<td>$12,500</td>
<td>$12,500</td>
<td>$12,500</td>
<td>$12,500</td>
<td>$12,500</td>
</tr>
</tbody>
</table>

PV of $1 at i = 12% is calculated as follows:

<table>
<thead>
<tr>
<th>Payment</th>
<th>PV of $1 at i = 12%</th>
<th>PV</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$12,500</td>
<td>$8,897</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>$12,500</td>
<td>$7,944</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>$16,841</strong></td>
<td></td>
</tr>
</tbody>
</table>

Cost Today? = $16,841
End of Chapter 6